VORTEX EFFECT AS A CONSEQUENCE OF NEGATIVE TURBULENT DIFFUSIVITY AND VISCOSITY

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Conditions of the appearance of negative turbulent viscosity and diffusivity have been analyzed. The vortex effect has been explained on the basis of negative turbulent diffusivity as a result of the reverse of the cascade of turbulent energy transfer.

As is known, the vortex effect was discovered by Ranque in 1931. Its essence lies in the energy separation of a strongly swirling turbulent flow of a compressible gas into hot and cold components. Near-axial layers are cooled and peripheral layers are heated. Here, cold and hot flows are led off in opposite directions. In monograph [1], a review is given of a few dozen of theoretical works in which attempts were made to describe the vortex effect. In the current study, it is proposed that it should be explained from the viewpoint of the reverse of the cascade transfer of turbulent energy. This implies that turbulent viscosity can assume negative values [2] and energy is transferred from a random pulsatory motion to an ordered mean one. In some works [3, 4], it has been shown that the effect of negative turbulent viscosity manifests itself in the case of two-dimensional character of turbulence and is attributed to the fact that energy transfer from smaller scales to larger ones is compensated by the enstrophy flow (half the square of vorticity) in the opposite direction. However, some studies where three-dimensional turbulent flows have been analyzed numerically (using proper orthogonal decomposition (POD) in [5] and direct numerical simulation (DNS) in [6]]) demonstrated that negative turbulent viscosity can also manifest itself in three-dimensional flows, although these processes are here less clearly expressed. A theoretical possibility of the effect of negative viscosity has been substantiated in [7], where the approximation of the renormalization group (RNG) approach was used.

In [8], based on the RNG approach, an expression for turbulent viscosity was obtained:

$$\nu_t \frac{d\nu_t}{dk} = -\frac{E}{b_d k^2}\,, \tag{1}$$

where $b_2 = 4$ for two-dimensional turbulence and $b_3 = 5$ for a three-dimensional one. With the Kolmogorov spectrum of turbulent energy

$$E = C_K k^{-5/3} \varepsilon^{2/3} \tag{2}$$

Eq. (1) admits both positive and negative values of turbulent viscosity. In the same work, the differential equation

$$\frac{\varepsilon}{E}\frac{dE}{dk} + 2k^2 \nu_t E = A_{ext}(k)\,, \tag{3}$$

which describes the distribution of the energy spectrum of turbulence over the inertial range, was derived. With no work of external forces, $A_{ext}(k) = 0$, the solution of the system of equations (1) and (3) for the energy spectrum gives expression (2), and for turbulent viscosity,

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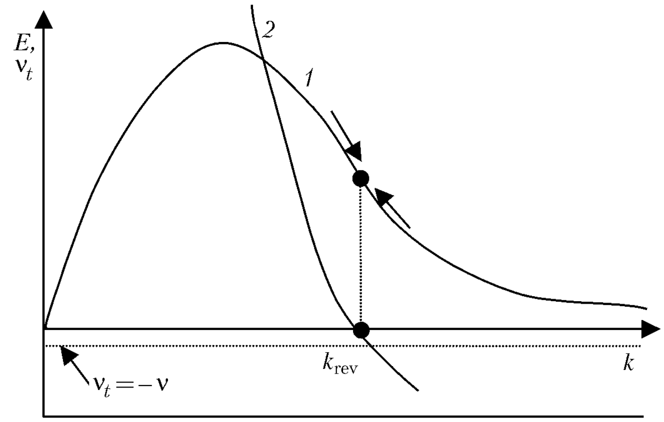

Fig. 1. Distributions of the energy spectrum (1) and turbulent viscosity (2).

$$\nu_t = \frac{5}{6C_K}\varepsilon^{1/3}k^{-4/3} \tag{4}$$

in the case of two- and three-dimensional turbulence alike.

Numerical investigations showed [9] that with the action of external forces where the reverse cascade mechanism of energy transfer is realized the shape of energy spectrum (2) in the inertial range does not change. This makes it possible to obtain an expression for turbulent viscosity based on Eq. (3) with allowance for Eq. (2):

$$\nu_t = \frac{5}{6C_K}\varepsilon^{1/3}k^{-4/3} + \frac{A_{ext}(k)}{2C_K}\varepsilon^{-2/3}k^{-1/3} . \tag{5}$$

Since the work of external forces can be regarded as the work done on a system, it has a negative value [10]. Consequently, Eq. (5) allows one to predict the appearance of negative turbulent viscosity at a certain intensity of the work of external forces. In the case with no external forces, Eq. (5) converts into Eq. (4). If this work is supplied to the system in a certain portion of the wave-number spectrum Δk

$$A_{ext}(k) = \frac{\eta}{\Delta k} ,$$

Eq. (5) takes the form

$$\nu_t = \frac{5}{6C_K}\varepsilon^{1/3}k^{-4/3} - \frac{\eta}{2C_K\Delta k}\varepsilon^{-2/3}k^{-1/3} . \tag{6}$$

From the latter relation it follows that turbulent viscosity assumes negative values under the condition

$$k_{rev} = k > \frac{5\varepsilon\Delta k}{3\eta} .$$

Hence it is seen that the range of existence of negative turbulent viscosity (of the reverse Richardson mechanism) depends on the relation between the rates of energy dissipation and energy supply — the higher the rate of energy supply and the lower the rate of energy dissipation, the wider the range in which reverse transfer of turbulent energy is effected. Furthermore, the narrower the interval of the supply of external energy η = idem, the wider the zone of negative turbulent viscosity. Thus, there are two ranges in the wave space, viz., those of direct and reverse transfer of turbulent energy (see Fig. 1). The boundary of existence of these ranges is the point $k = k_{rev}$. When $k > k_{rev}$, the reverse mechanism is prevalent and energy is transferred from smaller vortices to larger ones, although the possibility is not excluded that a direct cascade accompanied by energy dissipation might simultaneously be realized in this range.

However, it does not manifest itself clearly against the background of the reverse cascade. When $k < k_{rev}$, only direct energy transfer is realized. Therefore, energy transferred from smaller scales to larger ones reaches the region $k = k_{rev}$ and afterward, on passing the zone $k \cup [0, k_{rev}]$, is transferred to the main flow. Thus, energy transfer is of a non-local [11], hysteresis character.

Equation (6) can be rearranged using the RNG approach [12]. For this, by eliminating from expression (6) the wave number based on expression (9), we obtain

$$\nu_t = \frac{10K^2}{27C_K^2\varepsilon} - \frac{\sqrt{K}\,\eta}{\sqrt{6}\,C_K^{3/2}\varepsilon\Delta k}.$$

We next proceed to the analysis of processes in the fields of centrifugal mass forces. For this, we write balance equations for turbulent kinetic energy in the equilibrium approximation, i.e., disregarding advection, gradient (molecular) diffusion, and turbulent diffusion of pressure energy and kinetic energy, in polar coordinates and with allowance for the azimuthal symmetry [13]

$$\underbrace{-\overline{u'v'}\left(\frac{\partial U}{\partial r}-\frac{U}{r}\right)}_{\mathrm{I}} - \underbrace{(\overline{u'^2}-\overline{v'^2})\frac{V}{r}}_{\mathrm{II}} = \underbrace{\frac{\tau}{\rho}\left(\frac{\partial U}{\partial r}-\frac{U}{r}\right)}_{\mathrm{I}} - \underbrace{(\overline{u'^2}-\overline{v'^2})\frac{V}{r}}_{\mathrm{II}} = \underbrace{\varepsilon}_{\mathrm{III}}. \quad (7)$$

In Eq. (7), term I describes the turbulence generation due to energy taken up from the main flow, term II defines the action of centrifugal forces on the flow, and term III accounts for dissipation. Equation (7) is written in the two-dimensional approximation, since it was noted above that negative viscosity is characteristic of two-dimensional processes. Let us analyze this equation. Several variants are possible. Suppose terms I and II are positive (positive turbulent viscosity). Naturally, this situation is possible when term II is larger in magnitude than term I such that their difference remains positive, as required by Eq. (7). This means that turbulent energy is generated by the main flow and centrifugal forces exert a conservative effect, suppressing turbulence. In the next variant, term I is positive (positive eddy viscosity) and term II is negative. Therefore, the right-hand side of Eq. (7) remains positive at any magnitudes of terms I and II. This implies that turbulence is generated by both the main flow (positive turbulent viscosity) and the work of centrifugal forces (an active effect of centrifugal forces). Finally, there is the third variant where terms I and II are negative (negative turbulent viscosity), but term II is larger in magnitude than term I. This point is interpreted as follows: energy is transferred to turbulent pulsations due to the work of centrifugal forces. A part of this energy is transferred by the reverse cascade to the main flow and a part is dissipated.

Term II in Eq. (7) can have negative values (an active effect of centrifugal forces) in two cases, where the radial component of turbulent pulsations is (1) larger than the azimuthal (tangential) component at a positive value of the average radial viscosity and (2) smaller than the azimuthal component at a negative value of the average radial velocity. As experimental investigations show, a situation arises rather frequently where the radial component of turbulent pulsations has the largest value in comparison with other components of the pulsation field in swirling flows (see, for example, [14]). Negative values of the average radial velocity are realized is stable macrovortex structures when the magnitudes of this velocity component decrease from the periphery of a macrovortex to its center. Such a velocity distribution is observed in tornado natural formations. Here, from the continuity equation it follows that the axial velocity is intensified along the axis of a macrovortex. As is seen from Eq. (7), the general condition of manifestation of the influence of centrifugal forces is turbulence anisotropy. Evidently, this condition is fulfilled automatically in the fields of centrifugal forces.

Expressing the work of centrifugal forces from Eq. (7), which they perform on turbulence, and substituting it into the expressions for turbulent viscosity, we obtain

$$\nu_t = \frac{5}{6C_K}\varepsilon^{1/3}k^{-4/3} + \frac{\overline{u'^2}(k) - \overline{v'^2}(k)}{2C_K}V\varepsilon^{-2/3}k^{-1/3}. \quad (8)$$

In order to represent components of turbulent pulsations in the Fourier space (in the wave space), the RNG approach [12] is used, according to which

$$\left.\begin{matrix}\overline{u'^2}(k)\\ \overline{v'^2}(k)\end{matrix}\right\} \sim \frac{S_d}{2(2\pi)^{d+1}} \int_k^{\infty} k^{d-1} \int_{-\infty}^{\infty} \mathrm{Tr}\left(\frac{\overline{W_m(\mathbf{k},\omega)\,W_n(-\mathbf{k},-\omega)}}{(2\pi)^{d+1}\,\delta(\mathbf{k}+\mathbf{k}')\,\delta(\omega+\omega')}\right) d\omega dk\,,$$

where $S_d = 2\pi^{d/2}/\Gamma(d/2)$. Details of calculating this integral are given in [15]. The problem is that the RNG approach disregards turbulence anisotropy. Therefore, the calculated results can be represented in the following form:

$$\overline{v'^2}(k) - \overline{u'^2}(k) = \alpha(\Omega)\,K = \alpha(\Omega)\,\frac{3}{2}\,C_K \varepsilon^{2/3} k^{-2/3}\,. \tag{9}$$

In substituting Eq. (9) into Eq. (8), it should be taken into account that flows in the fields of centrifugal forces are of an anisotropic character and the degree of anisotropy can be determined by the intensity of the action of centrifugal forces $\alpha(\Omega)$. As a result, we obtain

$$\nu_t = \frac{5}{6C_K}\,\varepsilon^{1/3} k^{-4/3} - \frac{3\alpha(\Omega)}{4k}\,V = \nu_{t0} - \frac{3\alpha(\Omega)}{4k}\,V\,. \tag{10}$$

Expression (10) indicates that the origination of the reverse cascade (of negative viscosity) is determined by the magnitude and sign of the second term on the right-hand side. As was to be expected, the magnitude of this term is independent of the dissipation rate but is determined only by the intensity of the centrifugal forces. Furthermore, from expression (10) it follows that with a corresponding sign of the second term the region of negative viscosity manifests itself in the range

$$k_{\mathrm{rev}} = k > \frac{1000\varepsilon}{729\,(C_K \alpha(\Omega)\,|V|)^3}\,. \tag{11}$$

Clearly, the width of this range is proportional to the dissipation rate ε and inversely proportional to the work of centrifugal forces $\alpha(\Omega)V$.

Eliminating from expression (10) the wave number based on expression (9) gives

$$\nu_t = \frac{10K^2}{27C_K^3\varepsilon} - \frac{\alpha(\Omega)\,K^{3/2}}{\sqrt{6}\,C_K^{3/2}\varepsilon}\,V\,. \tag{12}$$

From the latter expression it follows that the condition of the appearance of negative viscosity is of the form

$$K < \frac{243}{200}\,C_K^3 \alpha(\Omega)\,V^2\,. \tag{13}$$

Relation (13) indicates that, in turbulent flows, local zones emerge where the reverse of the Richardson cascade mechanism of turbulent energy transfer occurs and energy goes from turbulent pulsations to the mean flow. In these zones, energy supplied to turbulent structures due to the work of external forces in the wave-number range, which is defined by inequality (11), should be larger than the threshold value of kinetic energy of flow turbulence (13). In this case, an excess of turbulent energy, as it were, arises in a local zone, which precisely is transformed into energyof the main flow.

The RNG approach gives the following differential equation for the effective (total) Prandtl number:

$$\frac{d\mathrm{Pr}_\Sigma^{-1}}{d\nu_\Sigma} = \frac{1}{\nu_\Sigma}\left(\frac{d-1}{d}\,\tilde{A}_d^{-1}\,\frac{1}{1+\mathrm{Pr}_\Sigma^{-1}} - \mathrm{Pr}_\Sigma^{-1}\right).$$

The solution of this equation for the d-dimensional space [11] is

$$\left|\frac{\mathrm{Pr}_\Sigma^{-1}-\xi}{\mathrm{Pr}^{-1}-\xi}\right|^{1+\xi}\left|\frac{\mathrm{Pr}_\Sigma^{-1}+\xi}{\mathrm{Pr}^{-1}+\xi}\right|^{\xi}=\left|\frac{\nu}{\nu_t+\nu}\right|^{2\xi+1}, \tag{14}$$

where

$$\xi=\frac{1}{2}\left(\sqrt{1+4\,\frac{d-1}{d}\,\tilde{A}_d^{-1}}-1\right);\quad \tilde{A}_d=\frac{d-1}{2\,(d+2)}\,.$$

With the condition $\left|\nu/\nu_t\right|\to 0$, from Eq. (14) it follows that $\mathrm{Pr}_\Sigma^{-1}\to\xi$. Then, introducing the variable $x=\mathrm{Pr}_\Sigma^{-1}-\xi$, it is possible to write the approximate solution of Eq. (14):

$$x\approx\left|\mathrm{Pr}^{-1}-\xi\right|\left|\frac{1+2\xi}{1+\mathrm{Pr}^{-1}+\xi}\right|^{-\frac{\xi}{2\xi+1}}\left|\frac{\nu}{\nu_t+\nu}\right|^{\frac{2\xi+1}{\xi+1}}.$$

Hence follows the expression for turbulent diffusivity

$$a_t\approx\nu_t\left(\xi+\left|\mathrm{Pr}^{-1}-\xi\right|\left|\frac{1+2\xi}{1+\mathrm{Pr}^{-1}+\xi}\right|^{-\frac{\xi}{2\xi+1}}\left|\frac{\nu}{\nu_t+\nu}\right|^{\frac{2\xi+1}{\xi+1}}\right)-a\,. \tag{15}$$

In the zones where $\nu_t=-\nu$, which are evidently close to the regions of reversal of the sign of turbulent viscosity, the approximate solution of Eq. (14) assumes the form

$$a_t\approx a\left(\mathrm{Pr}\left|\mathrm{Pr}^{-1}-\xi\right|^{\frac{\xi+1}{2\xi+1}}\left|1+\mathrm{Pr}^{-1}+\xi\right|^{\frac{\xi}{2\xi+1}}-1\right). \tag{16}$$

Equations (15) and (16) indicate the possibility of the existence of negative turbulent diffusivity. In this case, heat is transferred from a less heated medium to a more heated one like momentum is transferred against the velocity "gradient" at negative turbulent viscosity. Evidently, such a mechanism is realized with the vortex effect when a gas of a turbulent, compressed, strongly swirling flow becomes separated. Here, the work of centrifugal forces is transformed into turbulent pulsations that thereafter transfer energy to the main flow, i.e., the reverse cascade of turbulent energy transfer is realized, which is characteristic of processes with negative turbulent viscosity. The same reverse cascade causes the effect of negative turbulent diffusivity, which precisely leads to the gas separation into cold and warm "phases." Such phenomena where heat is transferred against the temperature gradient occur in atmospheric and oceanic processes [2].

NOTATION

A_{ext}, specific work of external forces, m^3/sec^3; $\tilde{A}_d$, dimensionless coefficient dependent on the space dimensionality; a, diffusivity, m^2/sec; a_t, turbulent diffusivity, m^2/sec; d, space dimensionality; C_K, Kolmogorov constant; E, spectrum of turbulent kinetic energy, m^3/sec^2; K, turbulent kinetic energy, m^2/sec^2; k, wave number, m^{-1}; $\mathbf{k}$, wave-number vector, m^{-1}; Pr, Prandtl number; Pr_Σ, effective (total) Prandtl number, which is the ratio of total (molecular + turbulent) viscosity to total (molecular + turbulent) diffusivity; r, radial coordinate, m; S_d, surface area of the d-dimensional sphere; Tr, trace of the matrix; W, Fourier transform of velocity, m^{d+1}; U, average tangential velocity, m/sec; u', pulsatory component of tangential velocity, m/sec; V, average radial velocity, m/sec; v', pulsatory component of radial velocity, m/sec; $x=\mathrm{Pr}_\Sigma^{-1}-\xi$, dimensionless parameter dependent on the space dimensionality; α, coefficient of anisotropy of turbulence; Γ, gamma function; δ, delta function; Δk, portion of the wave-number spectrum in which the work is supplied to the system, m^{-1}; ε, dissipation rate, m^2/sec^3; η, rate of the energy supply to turbulent pulsations m^2/sec^3; ν, kinematic molecular viscosity, m^2/sec; ν_t, kinematic turbulent viscosity, m^2/sec; ν_Σ, effective (total) viscosity (molecular + turbulent), m^2/sec; ξ, dimensionless parameter; ρ, density, kg/m^3; τ, shear stress, Pa; ω, frequency, sec^{-1}; Ω, angular velocity of rotation of the swirling flow, sec^{-1}. Subscripts: ext, external forces; m, n, velocity component of the d-dimensional space; rev, bound of the range of the Richardson reverse mechanism; t, turbulent; 0, absence of mass forces.

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